# DERIVATIVES (examples - part II )

In this part we try to explain search of derivatives for complex functions.

First to clarify when function is complex?

It is, most said, every function that is not in table of derivatives( there are only elementary functions).

**Here are a few examples:**

**1. Find derivative of function** $y = (1+5x)^{12}$

**Solution:**

How do we think?

If we have simple function $y = x^{12}$ derivative would be $y` = 12x^{11}$ and that would not be a problem.

But instead of x we have 1+5x, it tells us that the function is complex!

We do the same as in case of basic function, and add a derivative of "what" is complex.

So: $y = (1+5x)^{12}$

$y` = 12(1+5x)^{11}\mathbf{(1+5x)`}$

$y` = 12(1+5x)^{11}5$

$\mathbf{y` = 60(1+5x)^{11}}$

**2. Find derivative of function** $y = \sqrt{\sin x}$

**Solution:**

Remind yourself : in table of derivatives we have $y = \sqrt{x} \longrightarrow y` = \frac{1}{2\sqrt{x}}$ .

But, because "in" the root we have sinx, function is complex!

$$y = \sqrt{\sin x}$$

$$y` = \frac{1}{2\sqrt{\sin x}}\mathbf{(\sin x)`}$$

$$y` = \frac{1}{2\sqrt{\sin x}}\cos x$$

**3. Find derivative for** $y = e^{x^2+2x-3}$

**Solution:**

We know that $(e^x)` = e^x$ but instead of x we have $x^2 + 2x - 3$, and it means that we have complex function.

$$y = e^{x^2+2x-3}$$

$$y` = e^{x^2+2x-3}(x^2 + 2x - 3)`$$

$$y` = e^{x^2+2x-3}(2x + 2)$$

**4. Find derivative for** $y = \ln\frac{1+x}{1-x}$

**Solution:**

$$y = \ln\frac{1+x}{1-x}$$

$$y` = \frac{1}{\frac{1+x}{1-x}}\left(\frac{1+x}{1-x}\right)` \longrightarrow \left(\frac{1+x}{1-x}\right)` \text{ quotient rule}$$

$$y` = \frac{1}{\frac{1+x}{1-x}}\frac{(1+x)`(1-x) - (1-x)`(1+x)}{(1-x)^2}$$

$$y` = \frac{1}{1+x}\frac{1-x+1+x}{1-x}$$

$$y` = \frac{1}{1+x}\frac{2}{1-x}$$

$$y` = \frac{2}{1-x^2} \quad \text{final solution!}$$

Table of derivates for complex functions: If y = f(u) and u = g(x) then y` = f `(u) g`(x)

1. $(u^2)` = 2u\, u`$
2. $(u^n)` = nu^{n-1} u`$

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3. $(a^u)` = a^u \ln a\, u`$
4. $(e^u)` = e^u u`$
5. $(\log_a u)` = \frac{1}{u \ln a} u`$
6. $(\ln u)` = \frac{1}{u} u`$
7. $\left(\frac{1}{u}\right)` = -\frac{1}{u^2} u`$
8. $\sqrt{u}` = \frac{1}{2\sqrt{u}} u`$

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9. $(\sin u)` = \cos u\, u`$
10. $(\cos u)` = -\sin u\, u`$
11. $(\mathrm{tg}\, u)` = \frac{1}{\cos^2 u} u`$
12. $(\mathrm{ctg}\, u)` = -\frac{1}{\sin^2 u} u`$

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13. $(\arcsin u)` = \frac{1}{\sqrt{1-u^2}} u`$
14. $(\arccos u)` = -\frac{1}{\sqrt{1-u^2}} u`$
15. $(\mathrm{arctg}\, u)` = \frac{1}{1+u^2} u`$
16. $(\mathrm{arcctg}\, u)` = -\frac{1}{1+u^2} u`$

**5. Find derivative of functions:**

a) $y = \sin^5 x$

b) $y = \sin 5x$

**Solution:**

a) $y = \sin^5 x$

$y` = 5\sin^4 x(\sin x)`$

$y` = 5\sin^4 x \cdot \cos x$

b) $y = \sin 5x$

$y` = \cos 5x \cdot (5x)`$

$y` = \cos 5x \cdot 5 = 5\cos 5x$

**6. Find derivative of function:** $y = \ln\sqrt{\frac{1-\sin x}{1+\sin x}}$

**Solution:**

$$y = \ln\sqrt{\frac{1-\sin x}{1+\sin x}}$$

$$y` = \frac{1}{\sqrt{\frac{1-\sin x}{1+\sin x}}}\left(\sqrt{\frac{1-\sin x}{1+\sin x}}\right)`$$

**now we do** $\sqrt{u}` = \frac{1}{2\sqrt{u}}u`$ **where is u =** $\frac{1-\sin x}{1+\sin x}$

$$y` = \frac{1}{\sqrt{\frac{1-\sin x}{1+\sin x}}}\frac{1}{2\sqrt{\frac{1-\sin x}{1+\sin x}}}\left(\frac{1-\sin x}{1+\sin x}\right)`$$

$$y` = \frac{1}{2\frac{1-\sin x}{1+\sin x}}\frac{(1-\sin x)`(1+\sin x)-(1+\sin x)`(1-\sin x)}{(1+\sin x)^2}$$

$$y` = \frac{1}{2\frac{1-\sin x}{1+\sin x}}\frac{-\cos x(1+\sin x)-\cos x(1-\sin x)}{(1+\sin x)^2}$$

$$y` = \frac{1}{2\frac{1-\sin x}{1+\sin x}}\frac{-\cos x-\cos x\sin x-\cos x+\cos x\sin x}{(1+\sin x)^2}$$

$$y` = \frac{1}{2\frac{1-\sin x}{1+\sin x}} \frac{-2\cos x}{(1+\sin x)^2}$$

$$y` = \frac{-\cos x}{(1-\sin x)(1+\sin x)}$$

$$y` = \frac{-\cos x}{1-\sin^2 x}$$ **we will use : $\sin^2 x + \cos^2 x = 1$**

$$y` = \frac{-\cos x}{\cos^2 x}$$

$$y` = \frac{-1}{\cos\ \ x}$$ **final solution!**

**7. Find derivative of function: $y = \text{arc tg}\ \frac{1+x}{1-x}$**

**Solution:**

**We must work as** $(\text{arctgu})` = \frac{1}{1+u^2} u`$ where is $u = \frac{1+x}{1-x}$

**y = arc tg** $\frac{1+x}{1-x}$

$$y` = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \left(\frac{1+x}{1-x}\right)`$$

$$y` = \frac{1}{1+\frac{(1+x)^2}{(1-x)^2}} \frac{(1+x)`(1-x)-(1-x)`(1+x)}{(1-x)^2}$$

$$y` = \frac{1}{\frac{(1-x)^2+(1+x)^2}{(1-x)^2}} \frac{1(1-x)+1(1+x)}{(1-x)^2}$$

$$y` = \frac{1}{1-2x+x^2+1+2x+x^2} \frac{1-x+1+x}{1}$$ **simplify little...**

$$y` = \frac{2}{2+2x^2} = \frac{2}{2(1+x^2)} = \frac{1}{(1+x^2)}$$ **Therefore, the final solution is: $y` = \frac{1}{(1+x^2)}$**

**8. Find derivative for** **y = arcsin** $\frac{2x}{1+x^2}$

**Solution:**

**Work by the formula:** (arcsinu)\`= $\frac{1}{\sqrt{1-u^2}}u`$ where is u = $\frac{2x}{1+x^2}$

**y = arcsin** $\frac{2x}{1+x^2}$

$$y`=\frac{1}{\sqrt{1-(\frac{2x}{1+x^2})^2}}(\frac{2x}{1+x^2})`$$

$$y`=\frac{1}{\sqrt{1-\frac{4x^2}{(1+x^2)^2}}}\frac{2(1+x^2)-2x2x}{(1+x^2)^2}$$ **simplify...**

$$y`=\frac{1}{\sqrt{\frac{(1+x^2)^2-4x^2}{(1+x^2)^2}}}\frac{2+2x^2-4x^2}{(1+x^2)^2}$$

$$y`=\frac{1}{\sqrt{\frac{1+2x^2+x^4-4x^2}{(1+x^2)^2}}}\frac{2-2x^2}{(1+x^2)^2}$$

$$y`=\frac{1}{\sqrt{\frac{1-2x^2+x^4}{(1+x^2)^2}}}\frac{2(1-x^2)}{(1+x^2)^2}$$

$$y`=\frac{1}{\sqrt{\frac{(1-x^2)^2}{(1+x^2)^2}}}\frac{2(1-x^2)}{(1+x^2)^2}$$

$$y`=\frac{1}{\frac{1-x^2}{1+x^2}}\frac{2(1-x^2)}{(1+x^2)^2} \longrightarrow y`=\frac{2}{1+x^2}$$ **the final solution.**

**Derivatives - higher order**

y``= (y`)` ⟶ the second derivative is the first derivative of the first derivative

y```=(y``)` ⟶ the third derivative is the first derivative of the second derivative

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$y^{(n)}= (y^{n-1})$` ⟶ n-th derivative is the first derivative of (n-1)-th derivative

**So,here is practically no nothing new, because we actually always looking for the first derivative…**

**Here are a few examples:**

**Example 1.**

**Find the second derivative for the following functions:**

**a)** $y = 3x^2 - 4x + 5$

**b)** $y = e^{-x^2}$

**c)** $y = \dfrac{1+x}{1-x}$

**Solution:**

**a)** $y = 3x^2 - 4x + 5$

**y` = 6x – 4**

**y``= 6**

**b)** $y = e^{-x^2}$ **Watch out, this is a complex function ...**

$y$`$= e^{-x^2}(-x^2)$`$= e^{-x^2}$ (-2x) = -2x $e^{-x^2}$

y``= -2[x` $e^{-x^2}$ +( $e^{-x^2}$ )`x]

y``= -2[ $e^{-x^2}$ +(-2x $e^{-x^2}$ )x]

y``=-2[ $e^{-x^2}$ -2 $x^2$ $e^{-x^2}$ ]

y``= -2 $e^{-x^2}$ **[1-2 $x^2$ ]**

**c)** $y = \frac{1+x}{1-x}$

$$y` = \frac{(1+x)`(1-x) - (1-x)`(1+x)}{(1-x)^2}$$

$$y` = \frac{1(1-x) + 1(1+x)}{(1-x)^2}$$

$$y` = \frac{1 - x + 1 + x}{(1-x)^2}$$

$y` = \frac{2}{(1-x)^2}$ **now we are looking for y``, but we will write** $\frac{2}{(1-x)^2} = 2(1-x)^{-2}$

$$y` = 2(1-x)^{-2}$$
$$y`` = 2(-2)(1-x)^{-2-1}$$
$$y`` = -4(1-x)^{-3}$$
$$y`` = \frac{-4}{(1-x)^3}$$

**Example 2.**

We have function: **f(x)= $e^x$sinx.**

**Prove equality f ``(x) – 2f `(x) + 2f(x) = 0**

**Solution:**

**f(x)= $e^x$sinx**

**f `(x) = ($e^x$)`sinx + (sinx)`$e^x$**

**f `(x) = $e^x$sinx + cosx $e^x$**

**f ``(x) = ($e^x$sinx)` + (cosx $e^x$)`**

**f ``(x) = ($e^x$)`sinx + (sinx)`$e^x$ + (cosx)`$e^x$ + ($e^x$)`cosx**

**f ``(x) = $e^x$sinx + cosx $e^x$ - sinx $e^x$ + $e^x$cosx**

**f ``(x) = 2$e^x$cosx**

Now back in home equity:

**$f``(x) – 2f`(x) + 2f(x) =$**

**$2e^x cosx – 2 (e^x sinx + cosx\ e^x) + 2\ e^x sinx =$**

**$2e^x cosx – 2\ e^x sinx -2\ cosx\ e^x + 2\ e^x sinx =...=0$**

We prove that it is really : **$f``(x) – 2f`(x) + 2f(x) = 0$**